1.1 Modulus Functions

Question Paper

Course	CIE A Level Maths
Section	1. Algebra & Functions
Topic	1.1 Modulus Functions
Difficulty	Very Hard

Time allowed: 50

Score: /40

Percentage: /100

Question 1

State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $\text{f}: x \mapsto \tan x$

(ii) $\text{f}: x \mapsto \left|\frac{1}{x}\right|$

(iii) $\text{f}: x \mapsto \sqrt{x^2}$

(iv) $\text{f}: x \mapsto \pm\sqrt{25 - x^2}$

[4 marks]

Question 2

Solve the equation $|x^2 - 9| = 6 - 0.25x^2$, giving your answers in exact form.

[3 marks]

Question 3a

The functions $f(x)$, $g(x)$ are defined as follows

$$f(x) = |x^3 - 8| \qquad x \in \mathbb{R}$$
$$g(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = fg(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

[3 marks]

Question 3b

(b) There are between 0 and 4 solutions to the equation $fg(x) = c$, where c is a real number. Determine the values of c that produce each number of solutions.

[3 marks]

Question 4a

(a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{g}(x)|$ where

$$\mathrm{f}(x) = \sqrt{x} \qquad x \geq 0$$
$$\mathrm{g}(x) = 2x - 3 \qquad x \in \mathbb{R}$$

Label the points at which the graphs intersect the coordinate axes.

[3 marks]

Question 4b

(b) Solve the equation $\mathrm{f}(x) = |\mathrm{g}(x)|$.

[3 marks]

Question 4c

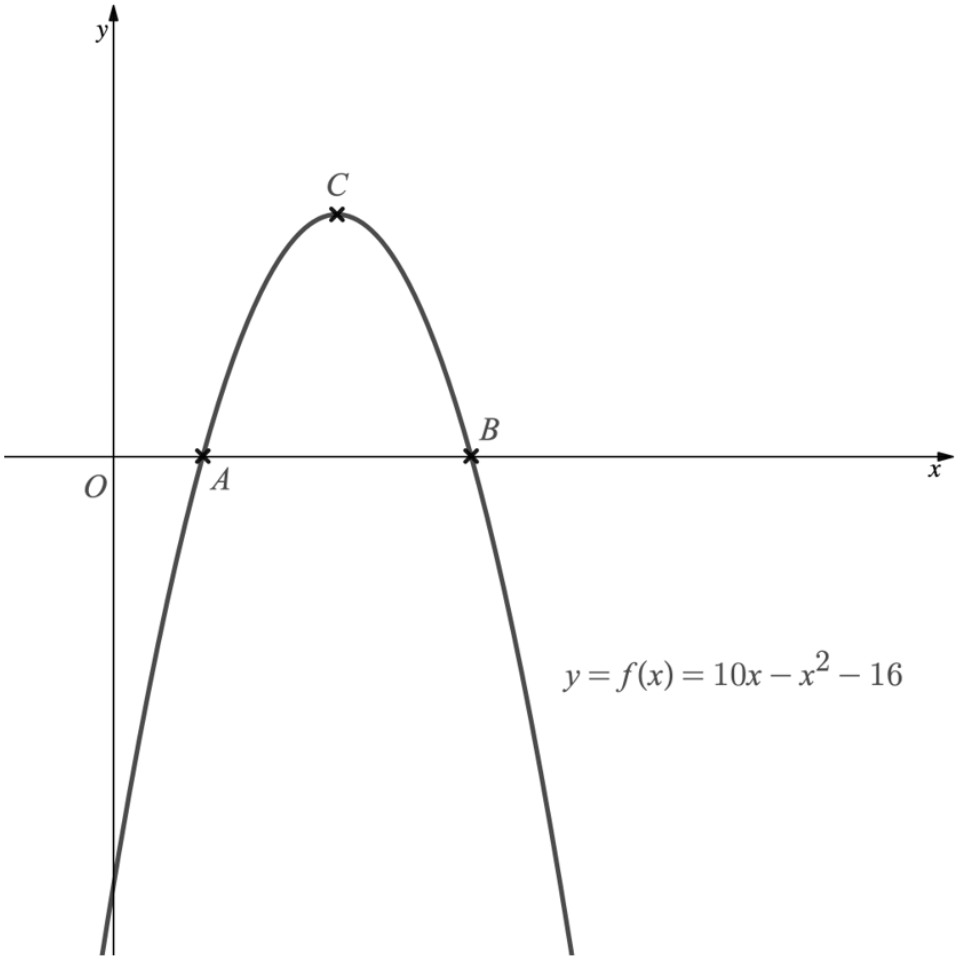
(c) Which of the solutions to $\mathrm{f}(x) = |\mathrm{g}(x)|$ is ***not*** a solution to $\mathrm{f}(x) = \mathrm{g}(x)$?

[1 mark]

Question 5a

A sketch of the graph with equation $y = f(x)$ where $f(x) = 10x - x^2 - 16$ is shown below.

Points A and B are the x-axis intercepts and point C is the maximum point on the graph.



(a) On the diagram above, sketch the graph of $y = -\left|\frac{1}{4}\mathrm{f}\left(\frac{1}{2}x\right)\right|$ labelling the image of the points A, B and C with A', B' and C'.

[3 marks]

Question 5b

(b) Show that the area of ABC is twice the area of triangle $A'B'C'$.

[4 marks]

Question 6

The function $f(x) = e^{3x} - x - 6$ is transformed by a sequence of transformations as described below.

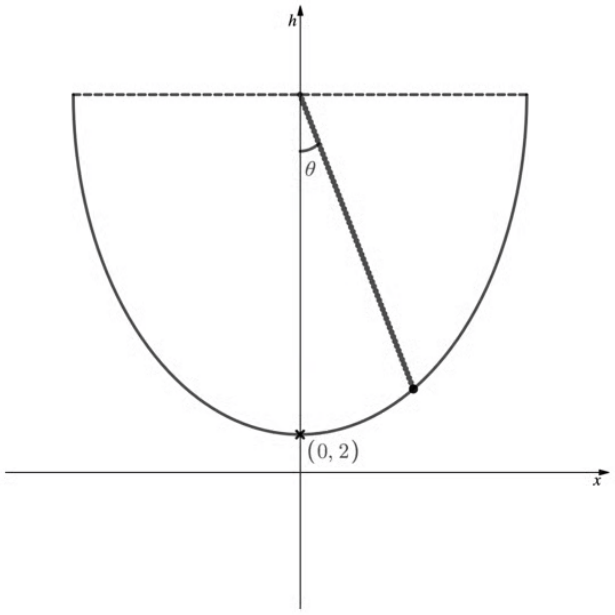
1. Horizontal stretch by scale factor 3,
2. The modulus of the function is then taken,
3. Reflection in the y-axis.

Write down the resulting transformation in terms of $f(x)$ as well as an expression in terms of x.

[4 marks]

Question 7a

A swing boat fairground ride is modelled as moving forwards and backwards along the path of a semi-circle, radius 18 m, as shown in the diagram below.



(a) Show that, for $0 \leq \theta \leq \frac{\pi}{2}$,

(i) the x-coordinate of the boat is given by $x = 18 \sin \theta$,

(ii) the y-coordinate is given by $y = 20 - 18 \cos \theta$.

[3 marks]

Question 7b

The model is refined so that the coordinates of the boat can be calculated from the time, t seconds, after the boat is set in motion. The x and y coordinates are now given by

$$x = 18\sin Bt \qquad\qquad y = 20 - 18|\cos Bt|$$

where B is a constant.

(b) (i) Briefly explain why the modulus of $\cos\theta$ is required for the y- coordinate.

(ii) Given that the time between the boat reaching its maximum height at either end of the ride is 8 seconds, find the value of B.

[3 marks]

Question 7c

(c) For $0 \leq t \leq 4$, find the times when the boat is equidistant from the ground and horizontally from the origin.

[3 marks]